DOCUMENT RESUME

ED 208 876 IR 009 790

AUTHOR Dugdale, Sharon
TITLE Green Globs: A Microcomputer Application for Graphing of Equations.
INSTITUTION Illinois Univ., Urbana. Computer-Based Education Research Lab.
SPONS AGENCY National Science Foundation, Washington, D.C.
REPORT NO CERL-R-E-21
PUB DATE Aug 81
GRANT SED80-12449
NOTE 22p.

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Algebra; Algorithms; *Computer Assisted Instruction; *Computer Graphics; Computer Programs; *Educational Games; *Graphs; Mathematics Instruction; *Microcomputers; Secondary Education; Trigonometry
IDENTIFIERS Equations (Mathematics)

ABSTRACT

This paper outlines the development of an activity that uses the computer's unique capabilities to provide students with a meaningful and highly motivating experience with the graphing of equations. The basic design of the game calls for the computer to display a coordinate grid on which it graphs any equations that are typed in by the student. Thirteen "Green Globs," each about .7 units in diameter, are scattered about the grid. The goal of the game is to explode all of the green globs by hitting them with graphs specified by typing in equations. If a shot misses the expected targets, diagnostic feedback is provided to debug the ideas used. The game is scored using an algorithm which encourages cleverly planned shots and provides a wide range of achievable scores. The decision to exclude the possibility of trigonometry functions in favor of other options is discussed. A provision of the game allows students to view any of the ten top scoring games and see what shots and strategies the top players have used. Highlights of classroom use of Green Globs are provided, including descriptions of techniques used by some of the more advanced students. One reference is cited. (CHC)

# Green Globs: A Microcomputer Application for Graphing of Equations

Sharon Dugdale

University of Illincis

Support for this work was provided by the National Science Foundation, (SED80-12449).

ability, coupled with the growing availability of low cost computers with sophisticated display capabilities, make computers a natural choice for materials intended to improve student's understanding of functions and graphs. This paper outlines the development of an activity that uses the computer's unique capabilities to provide students a meaningful and highly motivating experience with the graphing of equations.

## SOME DESIGN GOALS

Our observation during nine years of developing and testing computer-based materials has been that materials based on the inherently interesting aspects of mathematics produce far more powerful and satisfying learning experiences for students than the commonly seen programs that rely on flashy gimmicks unrelated to the mathematics. In creating this activity I sought to adhere to the following list of design principles. We have found these characteristics very desirable for instructional activities. Activities that have these characteristics we call "intrinsic models".

1. The student is given a working model to explore and manipulate.
2. The mathematics to be learned is intrinsic to the model.
3. The model provides a rich environment for exploration according to the student's own inclination and background.
4. Feedback is direct and meaningful, avoiding unnecessary verbiage and irrelevant gimmicks.
5. The mathematics is treated as inherently interesting, rather than hidden behind irrelevant themes.

ability, coupled with the growing availability of low cost computers with sophisticated display capabilities, make computers a natural choice for materials intended to improve student's understanding of functions and graphs. This paper outlines the development of an activity that uses the computer's unique capabilities to provide students a meaningful and highly motivating experience with the graphing of equations.

## SOME DESIGN GOALS

Our observation during nine years of developing and testing computer-based materials has been that materials based on the inherently interesting aspects of mathematics produce far more powerful and satisfying learning experiences for students than the commonly seen programs that rely on flashy gimmicks unrelated to the mathematics. In creating this activity I sought to adhere to the following list of design principles. We have found these characteristics very desirable for instructional activities. Activities that have these characteristics we call "intrinsic models".

1. The student is given a working model to explore and manipulate.
2. The mathematics to be learned is intrinsic to the model.
3. The model provides a rich environment for exploration according to the student's own inclination and background.
4. Feedback is direct and meaningful, avoiding unnecessary verbiage and irrelevant gimmicks.
5. The mathematics is treated as inherently interesting, rather than hidden behind irrelevant themes.

6. The activity is designed to be engaged in by students of widely varying background and ability in mathematics, but students find that the more math they apply, the better they do.
7. Rules are simple -- never more complex than the mathematics to be learned.
8. Feedback is diagnostic, so that students can tell at a glance what the error was and how it relates to a correct solution.

In short, the goal was to provide students a rich, mathematically accurate environment and the motivation to manipulate that environment to learn about graphs of equations.

## THE BASIC DESIGN

The computer displays a coordinate grid, on which it graphs any equations that are typed in by the student. Thirteen green objects, each about .7 units in diameter, are scattered about the grid, hence the name of the activity, "Green Globs".

The goal of the game is to explode all of the green globs by hitting them with graphs, specified by typing in equations. If a student's shot misses the expected targets, the graphic feedback (display of the student's graph) gives diagnostic information needed for the student to debug his or her ideas about graphs. Perhaps the graph was too wide, or too steep, or upside down. The student can test his or her diagnosis immediately on the next shot. Fig. 1 shows a sequence of displays from the game.

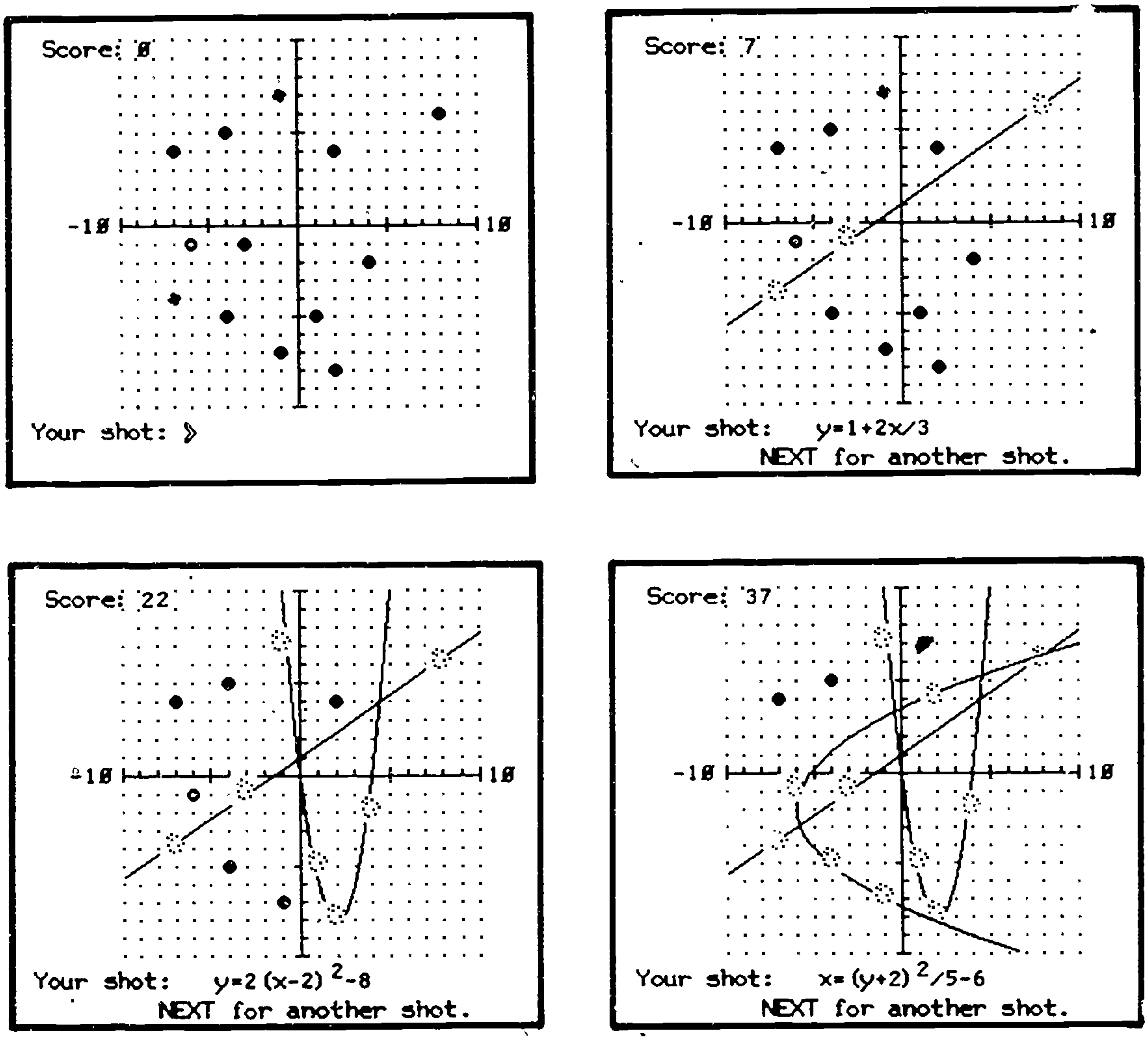



Fig. 1. Sequential displays from a game of Green Globs. The student types in equations, which are graphed by the computer. The green globs explode as they are hit by the graphs. Shown is the initial display of 13 globs, followed by the student's first three shots.

# SOME CRITICAL DESIGN DECISIONS

As in any creative effort, the designing of Green Globs involved many decisions -- far too many to detail here. In order to give some idea of the kinds of decisions that were necessary and the issues that were involved, I will discuss some critical decisions in two areas: how score is kept and what functions are allowed.

## Game Score

Originally, the goal of the game was to hit all of the globs in as few shots as possible. Observation of students revealed that nearly any serious player could hit all 13 globs with six shots, but few could hope to take fewer than four shots without a very lucky arrangement of globs. This narrow range of scores (with nearly all games taking four, five, or six shots) meant that students could work diligently at improving their games without experiencing any appreciable increase in success. It was necessary to change the scoring algorithm to better encourage cleverly planned shots and at the same time provide a wider range of achievable scores.

The following scoring algorithm, designed by my colleague, David Kibbey, has worked out much better. The student's score accumulates as follows: For each shot, the first glob hit gets one point, the second gets two points, the third gets four, and so on. For example, a five-glob shot would score 1+2+4+8+16, for a total of 31 points.

This scoring algorithm puts a higher premium on maximizing the number of globs hit with each individual shot. Now it is worth a lot to pick up one more glob on a shot. For example, the first four globs on a shot are worth 15 points, but adding a fifth glob will get 16 more points. Each added glob essentially doubles the score for the shot. This scoring algorithm has worked out very well with the students.

Zero Points for Zero Hits -- "No Penalty" Scoring

The new scoring algorithm raised the issue of what to do with "stray" shots that don't hit anything. Clearly, it is simple and mathematically consistent to score a no hit shot zero points. But one might question the fairness of having two players score the same when one makes several stray shots while the other hits something with each shot. Shouldn't we discourage carelessness by having a penalty for shots that hit nothing?

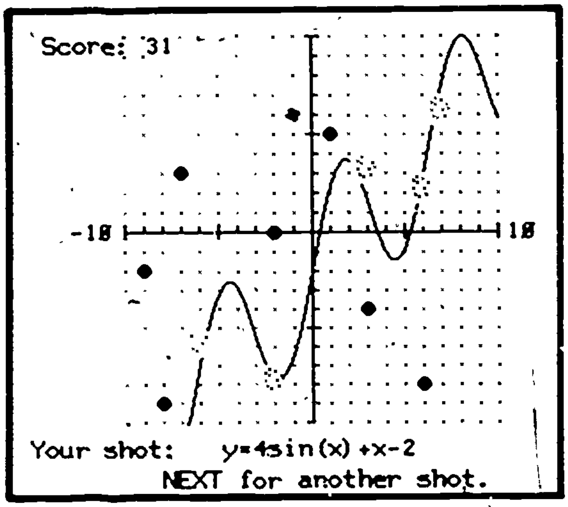
Consider the effect of this on students of various playing skill. A penalty for stray shots could discourage less able students from trying, and could discourage the somewhat more able students from experimenting with new techniques.

On the other hand, the more skilled the student becomes, the more likely he or she is to be able to include each glob in a shot with several others. Stray shots sometimes hit a glob, but usually not more than one. To the more skill'd player, accidentally hitting a single glob with a stray shot means that that particular glob cannot be included in a better planned shot, where it could have been worth far more than the one point it earned on the stray shot. So the better a player is, the more important it becomes to avoid mistakes.

Careful consideration led to a clear-cut decision to score no hit shots as zero points. The "no penalty" scoring encourages exploration by the less able players without encouraging carelessness by the more able players. Thus, the scoring algorithm as a whole provides appropriate challenge to a wide range of players without any special adjustments for individual players -- all are treated the same.

## Trig Functions and Other Options

The possibility of making trig functions allowable raised an interesting problem. As shown in fig. 2, functions like sine and cosine could broaden the scope of the activity, but a little student exploration could lead to use of functions like y=10sin(5x), which would reliably wipe out all of the globs in one shot. It was with some reluctance that I decided not to include trig functions. Trig functions would not only have provided an additional interesting class of functions to work with, but it could have been such a marvelously rewarding experience for the student who first worked out a "trig wipeout" function, like y=10sin(5x).



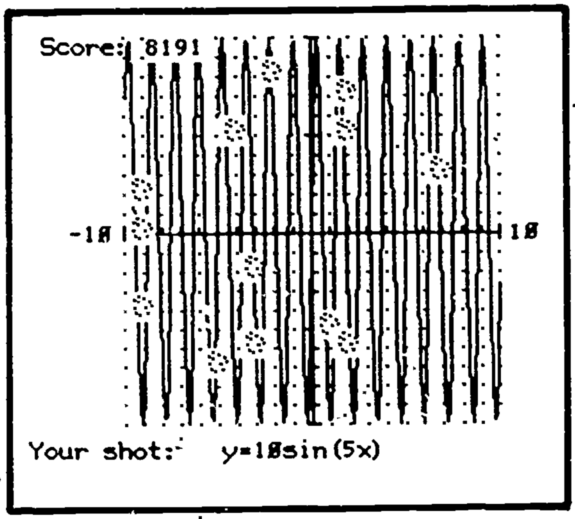




Fig. 2. The first frame shows an interesting use of a trig function. The application in the second frame is also interesting, but unfortunately it is an easily copied trick that reliably hits all of the globs in one shot, thus taking the challenge out of the game.

A similar marvelous experience is, however, available in polynomials, and it is presented here to illustrate how it contrasts with the previous case.

Algebra students are exposed to the fact that the expression (x-1)(x-3) will equal 0 when x is either 1 or 3, and hence that the graph of y=(x-1)(x-3) will cross the x axis at 1 and again at 3. By applying this technique, the student can construct a polynomial to cross the x axis at any desired points. Further, if some large number is also multiplied in to stretch the graph vertically, then the effect in the range plotted on the screen is essentially a vertical line through each chosen point. (See fig. 3.) So it is simple, using enough factors of the form (x-a), to construct a polynomial that will put a vertical line through any number of chosen points on the x axis, thus hitting all of the globs with one shot.

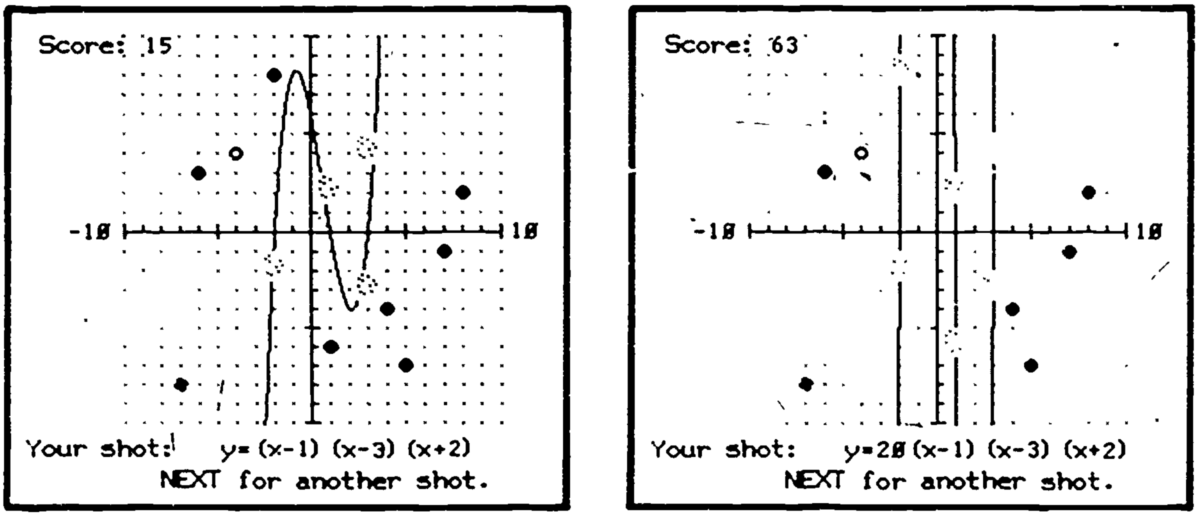



Fig. 3. In the first frame the student has constructed a function to cross the x axis at three chosen points. In the second frame, the student has multiplied this same function by 20 to stretch it vertically until what appears on the screen is essentially a series of vertical lines.

The catch, however, is that the space available for typing in an equation is limited by the edge of the screen. Only about 4 such factors will fit. This is where the "super polynomial" solution differs educationally from the "trig wipeout" solution discussed above. y=10sin(5x) is an easily copied (but not necessarily understood) equation that reliably hits all of the globs. In contrast, the "super polynomial" solution needs to be understood well enough for the student to choose appropriate factors for different games. This hopefully leads to a richer experience for the entire class, with more students figuring out how the "super polynomial" function works as they apply it to their own games.

It should be noted that discovery of the "super polynomial" is by no means the ultimate achievement, leaving nothing more to work on. The next breakthrough might, for example, come from a student who is including both (x-3) and (x+3) in the "super polynomial" but notes that (x-3)(x+3) equals $(x^2-9)$, which is much shorter, thus leaving room for more factors. This discovery might change the focus from deciding which individual vertical lines will maximize the score to, instead, which pairs of vertical lines could be used. Of course, horizontal lines are equally easily produced by writing x as a function of y, so the student will no doubt also be concerned with deciding which type of function would be better.

## A PROVISION FOR SHARING IDEAS AND STRATEGIES

The highest ten scores on each disk are stored in a "hall of fame". (See fig. 4.) Students' names are displayed along with their record-making scores. The records in the "hall of fame" change frequently as students achieve higher scores. Records with lower scores are automatically deleted to make room for newer higher scores.

Green Globs Top Ten Scores

| | | |
|---|---|---|
| 1. | 145 | Christy and Jeff |
| 2. | 138 | Keiran |
| 3. | 97 | Ted P. and Brian S. |
| 4. | 85 | Leslie, Sal, and Shannon |
| 5. | 82 | Ma?tha |
| 6. | 76 | Fred D., Barry, and Lori |
| 7. | 52 | Michaela and Pat |
| 8. | 48 | Darrell |
| 9. | 45 | Steven, Cynthia, and Craig |
| 10. | 44 | Dennis R. and Fred T. |

Press BACK to return to the game, or choose a record number (from 1 to 10) to see a replay of it. »

Fig. 4. The Green Globs Hall of Fame. Student's names are displayed besides their record-making scores. From this display, students can choose to see a "replay" of any of the top ten games.

A very important aspect of the activity is that all of the top ten games listed in the hall of fame are stored so that they can be viewed by other students who want to see what shots and strategies the top players have used. (See fig. 5.) Students are frequently observed to "replay" one or more of the record-making games to gather tools that could be useful to them in future games. This rich exchange of ideas and techniques leads far more students to see and try interesting applications of algebra than would be likely otherwise.

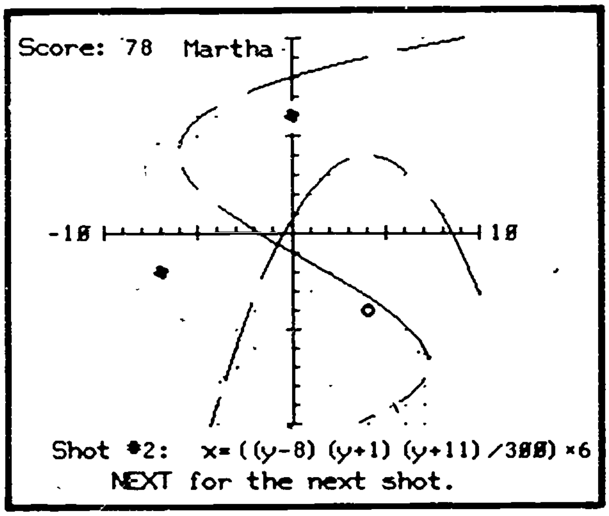



Fig. 5. A display from Martha's record making game. Martha's name is shown at the top of the display, and her equations are shown, one at a time, at the bottom, as they are graphed on the coordinate axes. Students can learn each other's strategies by watching shot-by-shot "replays" of games listed in the records. Can you figure out how Martha constructed the equation for her second shot?

## SOME HIGHLIGHTS OF CLASSROOM USE

Green Globs was first introduced into a classroom in February 1981. It was used throughout that spring semester, along with several related activities, in conjunction with a wide range of mathematics classes at Central and Centennial High Schools in Champaign, Illinois.

Usage ranged from classes that integrated the microcomputers into their work for several weeks, to classes who took two days to become familiar with the microcomputers and encourage students to continue using them outside of class. Students often used the microcomputers before or after school, during the noon hour, and at other times during the day when a class was not using them.

Green Globs was very popular with students of widely varying mathematical experience and abilities. With no background in algebra, students in the ninth grade general mathematics classes were quickly able to hit all of the globs with horizontal and vertical lines. Their early strategy was simply to first use those constant functions that would hit more than one glob. For example, if there were two globs on the vertical line x=4, the students were careful not to hit either of them with a horizontal shot.

Before long, though, these students began to notice what the more experienced students were doing. Then they wanted to know how to make lines with different slopes, and curves, too. They began replaying the algebra students' games, copying equations from those games, and trying the copied equations in their own games. Of course the globs in their own games were differently placed, but often the students could see that the graph of the copied equation would hit several globs if it could just be moved higher, or lower, or changed in some way. This led them to try different numbers in the equations to see how they could manipulate the graphs. It was fascinating to see these students, generally not at all fond of mathematics, so strongly motivated to work on concepts beyond what was required in their classwork.

Usage by students at all levels was predominantly by groups of 2 or 3 working together at each microcomputer, although students who wanted to work alone sometimes were able to do so. Some students, usually the more advanced ones, tended to become deeply engrossed in working out complicated or higher degree equations and seemed more intensely involved when working alone than would likely be possible in a group situation. These same students also analyzed other students' games through the replay option in the records section. It seems that each of these modes of use (individual and small group) provided different important learning and exploration opportunities. Is is interesting that the students we observed working in groups generally participated surprisingly equally. It was common to see students within the group frequently trading roles of advice giver and advice receiver.

After about six weeks of use, some students had been experimenting with polynomials in factored form, and the "super polynomial" emerged. While some students used this newly-discovered technique to achieve higher and higher scores, others found challenge in devising and using clever techniques to successfully compete for top scores by more elegant and original means.

One particularly elegant idea that showed up in the hall of fame is illustrated in figure 6. The student has constructed a parabola to hit several green globs, but has then added $1/(x-3.5)^2$ to the equation. The effect of this extra term is negligible for all values of x except those close to 3.5, so the resulting graph is the expected parabola everwhere except around x=3.5. When x gets close to 3.5, the denominator of the extra term approaches zero, making the function very large. Using this technique, the student is able to make the graph leave the parabolic path briefly to jump up and grab a few more globs.

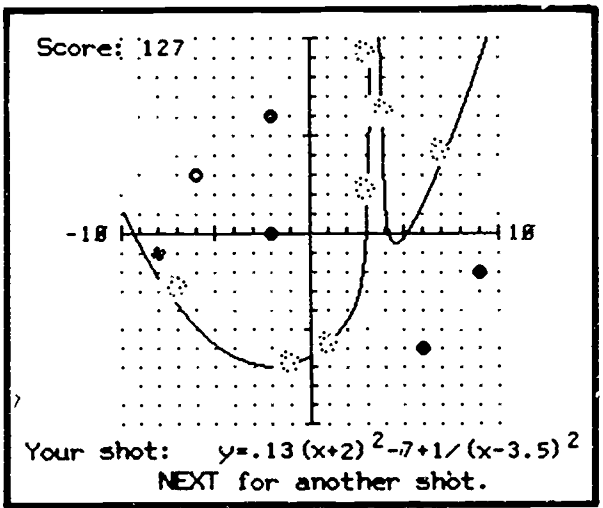



Fig. 6. The student has constructed a parabola to hit several green globs, but has then added $1/(x-3.5)^2$ to the function in order to make it jump up and capture three more globs near x=3.5.

Games using this trick evidenced some experimentation with the size of the exponent and the numerator. These could be manipulated to affect the distance between the two branches and the "tightness" of the curve where the special term began to take effect. Of course, the parity of the exponent also played a dramatic role, with even exponents sending both branches off the screen in the same direction and odd exponents making the graph exit from the screen in one direction and return from the opposite. Some students became surprisingly skillful at constructing graphs to do what they wanted!

One student became interested in higher degree equations of the form $y=a(x-h)^n+k$, which, with small coefficients and even exponents, produce graphs that look something like croquet wickets. (See fig. 7.) Possibly the most interesting aspect of this student's technique was that he used his pocket calculator to find the needed coefficients from the coordinates of the globs he intended to hit (not as trivial a task as it might seem, since a shot is rarely planned to hit globs dead center, but rather to touch as many globs as possible). It is gratifying to see students using technology to facilitate the tedious calculations, while experimenting with the larger concepts.

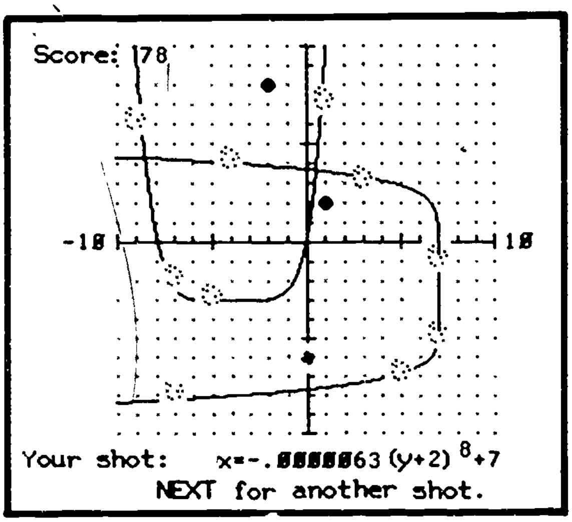



Fig. 7. Here a student is experimenting with equations of the form $y=a(x-h)^n+k$, where a is small and n is even, greater than 2. This student used a pocket calculator to find the necessary coefficients from the coordinates of the globs.

As an aside, I will note that another student set out to write a computer program to construct equations that would hit all of the globs. Unfortunately, this didn't work out as well.

At this point it seems worth mentioning one more design decision -- the positioning of the green globs. Early in the design process the question arose whether the globs should be carefully placed on particular graphs, rather than randomly scattered. This could encourage students to practice certain types of equations. However, since touching any part of a glob is sufficient to explode it, it is not really necessary to carefully place them. Each glob has a diameter of .7, and it is very likely that several of the thirteen globs can be hit with whatever types of equations the student is studying.

One argument for randomness is that it adds chance to the game, which is a motivating feature, especially for the less able players (those who need the practice most). The element of chance means that the student might get a more lucky arrangement of globs next game, and further, when the student does get a particularly nice arrangement, there is high motivation to avoid careless errors.

Perhaps the most convincing argument against arranging the globs in deliberate patterns is that it could greatly decrease the liklihood of students exploring beyond those particular types of equations. Given the inventive strategies students have already demonstrated during the few months they have played Green Globs, it would be unfortunate to restructure the activity in a way that might limit students' imaginations to whatever functions the author might choose to include. Certainly, any "rigging" of the game should be done with great care. Perhaps some of the globs could be carefully placed, leaving the rest randomly scattered. However, given that other activities in our sequence are designed to focus on particular graphs, it seems advantageous to place the globs randomly and let this activity foster as much creative innovation as possible.

## CONCLUSION

In order to realize the full instructional potential of the computer, we must go beyond its use for facilitating and simulating tasks that have traditionally been done in other ways. We must look toward developing its potential for classroom applications that were previously not possible (and therefore not even conceived), for which the computer is especially well suited.

Computers are capable of providing students carefully structured environments and motivations to explore and learn from those environments. In addition, computers offer students opportunities to easily manipulate complex aspects of these environments, plus the facility to communicate and share their ideas with large numbers of other students. Given such tools, students can (and do) create innovative problem solving strategies. The displaying of students' winning strategies for others to see enhances both the motivational and educational aspects of the activity.

Most important, such activities focus students' attention on their own substantial ability to use mathematics in interesting and creative ways to achieve what is to them a worthwhile goal.

## REFERENCE AND NOTES

National Assessment of Educational Progress, Mathematical Knowledge and Skills. Education Commission of the States, Denver, Colorado (August, 1979).

Green Globs is part of a prototype curriculum sequence which uses an intrinsic models approach to teach high school algebra. Funding for this work was provided by the National Science Foundation, NSF-SED 80-12449.

This and related activities are soon to be pubished for use on popular microcomputers. Details are available from the author.